

Matematika ta'lim yo'nalishi 2-kurs talabalari uchun "*Matematik analiz*" fanidan
yakuniy nazorat savollari

(2024-2025 o'y. 3-semestr)

Nazariy savollar

1. Ko'p o'zgaruvchili funksiya uchun Teylor formulasi.
2. Ko'p o'zgaruvchili funksiyaning ekstremum qiymatlari. Ekstremumning za'ruriy va yetarli sha'rtlari.
3. Oshkormas funksiyalar. Oshkormas funksiyalarning mavjud bo'lishi.
4. Oshkormas funksiyalarning uzluksizligi va differensiallanuvchiligi.
5. Sonli qator tushunchasi, uning yaqinlashuvchiligi.
6. Yaqinlashuvchi sonli qatorning xossalari.
7. Musbat hadli qatorlar va ularning yaqinlashish a'lomatlari.
8. Ixtiyoriy hadli qatorlar va ularning yaqinlashish a'lomatlari.
9. Absolyut yaqinlashuvchi qatorlar
10. Sha'rtli yaqinlashuvchi qatorlar. Riman teoremasi
11. Funksional ketma-ketliklarning tekis yaqinlashuvchiligi
12. Funksional qatorlarning tekis yaqinlashuvchiligi, Koshi kriteriyasi
13. Funksional qatorlarning tekis yaqinlashish alomatlari (Abel).
14. Funksional qatorlarning tekis yaqinlashish alomatlari (Veyershtrass).
15. Funksional qatorlarning tekis yaqinlashish alomatlari (Dirixle).
16. Funksional qatorlarning funksional xossalari (hadma-had limitga o'tish, qator yig'indisining uzluksizligi).
17. Funksional qatorlarning funksional xossalari (hadma-had integrallash va differensiallash).
18. Darajali qator. Darajali qatorlarning yaqinlashish radiusi va sohasi.
19. Darajali qator. Koshi-Adamar formulasi.
20. Darajali qatorlarning funksional xossalari.
21. Teylor qatori. Ba'zi bir elementar funksiyalarni Teylor qatoriga joyish.
22. Parametrga bog'liq xos integrallar va ularning funksional xossalari.
23. Parametrga bog'liq xos integrallar (umumiy hol).
24. Parametrga bog'liq xosmas integrallarning tekis yaqinlashuvchiligi
25. Tekis yaqinlashuvchi parametrga bog'liq xosmas integrallarning funksional xossalari.
26. Gamma va beta funksiyalar va ularning xossalari, ular orasidagi bog'liqlik

Amaliy masalalar

27. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{3^2}\right) + \dots + \left(\frac{1}{2^n} + \frac{1}{3^n}\right) + \dots$$

28. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

29. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)}$$

30. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\sum_{n=1}^{\infty} \ln \left(1 - \frac{2}{n(n+1)} \right)$$

31. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\sum_{n=1}^{\infty} \frac{1}{n!(n+2)}$$

32. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{(-1)^{n-1}}{2^{n-1}} + \dots$$

33. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} + \dots$$

34. Qatorni yaqinlashuvchilikka tekshiring: $\frac{1000}{1!} + \frac{1000^2}{2!} + \frac{1000^3}{3!} + \dots + \frac{1000^n}{n!} + \dots$

35. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} \frac{1}{2^n} \left(1 + \frac{1}{n} \right)^n$

36. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} \frac{n^{n+\frac{1}{n}}}{\left(n + \frac{1}{n} \right)^n}$

37. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} \frac{n! e^n}{n^{n+p}}$

38. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} \frac{1}{n^p} \sin \frac{\pi}{n}$

39. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+100}{3n+1} \right)^n$

40. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty} \frac{\ln^{100} n}{n} \sin \frac{\pi n}{4}$

41. Berilgan qatorning yaqinlashuvchi ekanligini ko'rsating va yig'indisini toping:

$$\sum_{n=1}^{\infty} \frac{\cos \frac{2\pi n}{3}}{2^n}$$

42. Qatorni yaqinlashuvchilikka tekshiring: $\sum_{n=2}^{\infty} \left(\frac{n-1}{n+1} \right)^{n(n-1)}$

43. $f(x, y) = 2x^2 - xy - y^2 - 6x - 3y + 5$ funksiyaning $n = 3$ bo'lgan holda $A(1, -2)$ nuqtasi atrofida Teylor formulasini yozing.

44. $f(x, y) = \sqrt{1 - x^2 - y^2}$ funksiyaning $n = 2$ bo'lgan holda $A(0, 0)$ nuqtasi atrofida

Teylor formulasini yozing.

45. $f(x, y) = e^x \sin y$ funksiyaning $n = 3$ bo'lgan holda $A(0, 0)$ nuqtasi atrofida Teylor formulasini yozing.

46. $f(x, y) = y^x$ funksiyaning $n = 2$ bo'lgan holda $A(1, 1)$ nuqtasi atrofida Teylor formulasini yozing.

47. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = e^{x-y}(x^2 - 2y^2)$

48. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = x^4 + y^4 + 2x^2y^2 - 8x + 8y$

49. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = e^{x^2-y}(5 - 2x + y)$

50. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = x^3 + y^3 - 3xy$

51. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = xy(1 - x - y)$

52. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = xy + \frac{50}{x} + \frac{20}{y} \quad (x > 0, y > 0)$

53. Berilgan funksiyaning ekstremumga tekshiring: $f(x, y) = x^2 + (y-1)^2$

54. Berilgan funksiyaning ko'rsatilgan D sohadagi eng katta va eng kichik qiymatlarini toping: $f(x, y) = x - 2y - 3$. $D = \{(x, y) \in R^2 : 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq x + y \leq 1\}$

55. Berilgan funksiyaning ko'rsatilgan D sohadagi eng katta va eng kichik qiymatlarini toping: $f(x, y) = 1 + x + 2y$. $D = \{(x, y) \in R^2 : x \geq 0, y \geq 0, x + y \leq 1\}$

56. Berilgan funksiyaning ko'rsatilgan D sohadagi eng katta va eng kichik qiymatlarini toping: $f(x, y) = x^2 + 3y^2 - x + 18y - 4$. $D = \{(x, y) \in R^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\}$

57. Berilgan funksiyaning ko'rsatilgan D sohadagi eng katta va eng kichik qiymatlarini toping: $f(x, y) = x^2 - y^2$. $D = \{(x, y) \in R^2 : x^2 + y^2 \leq 1\}$

58. Oshkormas ko'rinishda berilgan funksiyaning birinchi va ikkinchi ta'rtibli hosilalarini toping: $x^2 + 2xy - y^2 = a^2$

59. Oshkormas ko'rinishda berilgan funksiyaning birinchi va ikkinchi ta'rtibli hosilalarini toping: $\ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x}$

60. Oshkormas ko'rinishda berilgan funksiyaning birinchi va ikkinchi ta'rtibli hosilalarini toping: $y - \varepsilon \sin y = x \quad (0 < \varepsilon < 1)$

61. Funksional ketma-ketlikning berilgan to'plamdagi limit funksiyasini toping:

$$f_n(x) = x^n - 3x^{n+2} + 2x^{n+3}, \quad E = [0; 1]$$

62. Funksional ketma-ketlikning berilgan to'plamdagi limit funksiyasini toping:

$$f_n(x) = x^4 \cos \frac{1}{nx}, \quad E = (0; \infty)$$

63. Funksional ketma-ketlikning berilgan to'plamdagi limit funksiyasini toping:

$$f_n(x) = (x-1) \operatorname{arctg} x^n, \quad E = (0; \infty)$$

64. Funksional ketma-ketlikning berilgan to'plamdagi limit funksiyasini toping:

$$f_n(x) = \sqrt{x^2 + \frac{1}{\sqrt{n}}}, \quad E = \mathbb{R}$$

65. Funksional ketma-ketlikning berilgan to'plamda tekis yaqinlashuvchi ekanligini

ko'rsating $f_n(x) = e^{-nx^2}, \quad E = [1; \infty]$

66. Funksional ketma-ketlikning berilgan to'plamda tekis yaqinlashuvchi ekanligini

ko'rsating $f_n(x) = x^{2n}, \quad E = [0; \delta], 0 < \delta < 1.$

67. Funksional ketma-ketlikning berilgan to'plamda tekis yaqinlashuvchi ekanligini

ko'rsating $f_n(x) = \sqrt{n} \sin \frac{x}{n\sqrt{n}}, \quad E = \mathbb{R}$

68. Funksional qatorning yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} 2^n \sin \frac{x}{3^n}$

69. Funksional qatorning yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(x-2)^n}$

70. Funksional qatorning yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} n \sqrt[3]{\sin^n x}$

71. Funksional qatorning yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} \frac{x^n(1-x^n)}{n}$

72. Funksional qatorning absolyut yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{\pi x}{n}$

73. Funksional qatorning absolyut yaqinlashish sohasini toping: $\sum_{n=1}^{\infty} \frac{1}{n! x^n}$

74. Funksional qatorning berilgan sohada tekis yaqinlashuvchi ekanligini Veyershtrass a'lomati yordamida ko'rsating: $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$, $-1 \leq x \leq 1$.
75. Funksional qatorning berilgan sohada tekis yaqinlashuvchi ekanligini Veyershtrass a'lomati yordamida ko'rsating: $\sum_{n=1}^{\infty} \frac{1}{(x+n)^2}$, $0 \leq x < \infty$.
76. Funksional qatorning berilgan sohada tekis yaqinlashuvchi ekanligini Veyershtrass a'lomati yordamida ko'rsating: $\sum_{n=1}^{\infty} \frac{(-1)^n}{x+2^n}$, $-2 < x < +\infty$.
77. Darajali qatorning yaqinlashish radiusini toping: $\sum_{n=0}^{\infty} n^2 z^n$
78. Darajali qatorning yaqinlashish radiusini toping: $\sum_{n=0}^{\infty} 3^n (z+1)^n$
79. Darajali qatorning yaqinlashish radiusini toping: $\sum_{n=0}^{\infty} n! \left(\frac{z}{n}\right)^n$
80. Berilgan funktsiyani Makloren qatoriga yoying va yaqinlashish radiusini toping: e^{-x^2}
81. Berilgan funktsiyani Makloren qatoriga yoying va yaqinlashish radiusini toping:

$$\frac{x^2}{(1+x)^2}$$
82. Berilgan funktsiyani x_0 nuqtaning atrofida Teylor qatoriga yoying va yaqinlashish radiusini toping: $\frac{1}{x^2 - 5x + 6}$, $x_0 = 1$.
83. Berilgan funktsiyani x_0 nuqtaning atrofida Teylor qatoriga yoying va yaqinlashish radiusini toping: $\sin^3 x$, $x_0 = \frac{\pi}{4}$.
84. Berilgan funktsiyani x_0 nuqtaning atrofida Teylor qatoriga yoying va yaqinlashish radiusini toping: $\frac{1}{\sqrt{x^2 - 12x + 40}}$, $x_0 = 6$.
85. Limitni hisoblang: $\lim_{\alpha \rightarrow 0} \int_{-\pi}^{\pi} (1 + \cos \alpha x) e^{x \sin \alpha} dx$
86. Parametrga bog'liq integralni hisoblang: $I = \int_0^1 \frac{x^{\alpha_2} - x^{\alpha_1}}{\ln x} dx$, $0 < \alpha_1 \leq \alpha_2$.
87. Agar $I(\alpha) = \int_1^2 e^{\alpha x^2} \frac{dx}{x}$ bo'lsa $I'(\alpha)$ ni hisoblang.
88. $I(\alpha) = \int_0^1 \sin^2 \alpha x^2 dx$ funksiyasini R da uzluksiz ekanligini ko'rsating.

89. $I(\alpha) = \int_{-1}^2 \frac{x^2}{1+x^2+\alpha^2 x^4} dx$ funksiyasini R da uzluksiz ekanligini ko'rsating.

90. Limitni hisoblang: $\lim_{\alpha \rightarrow 0} \int_0^1 \sqrt{1+\alpha^2 x^4} dx$

91. Agar $I(\alpha) = \int_0^1 \sin(\alpha x) dx$ bo'lsa $I'(\alpha)$ ni hisoblang.